SARAJEVO JOURNAL OF MATHEMATICS DOI: 10.5644/SJM.17.02.02
Vol.17 (30), No.2 (2021), 143 – 150

THE LINE GRAPH OF A COMMUTING GRAPH ON THE DIHEDRAL GROUP D_{2n}

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ABSTRACT. Let Γ be a non-abelian group and $\alpha \subseteq \Gamma$. Then the Commuting graph $C(\Gamma,\alpha)$ has α as its vertex set and two distinct vertices in α are adjacent if they commute with each other in Γ. Let $G = L(C(\Gamma,\alpha))$ be the Line graph of the Commuting graph. A vertex v_i of G is given by $\{x,y\} = \{y,x\}$ where x and y are the vertices that are adjacent in $C(\Gamma,\alpha)$. In this paper, we discuss certain properties of the Line graph of the Commuting graph on the Dihedral group D_{2n}. More specifically, we obtain the chromatic number, clique number and genus of this graph.

1. INTRODUCTION

Let G be any graph. The *line graph* of G, denoted $L(G)$, is the graph whose points are the lines of G, with two points of $L(G)$ adjacent whenever the corresponding lines of G are adjacent. Various extensions of the concept of a line graph have been studied, including line graphs of line graphs, line graphs of multigraphs, line graphs of hypergraphs and line graphs of weighted graphs. For any integer $n \geq 3$, the Dihedral group D_{2n} is given by $D_{2n} = \langle r,s : s^2 = r^n = 1, rs = sr^{-1} \rangle$.

The line graph of $C(\Gamma,\alpha)$, denoted by $L(C(\Gamma,\alpha))$ has vertices as the lines of $C(\Gamma,\alpha)$, and two points of G are adjacent whenever the corresponding lines of $C(\Gamma,\alpha)$ are adjacent. We consider simple graphs which are undirected, with no loops and multiple edges.

A *graph* G consists of a finite nonempty set $V = V(G)$ of points together with a prescribed set E of unordered pairs of distinct points of V. Each pair $e = \{u,v\}$ of points in E is a line of G. We write $e = uv$ and say that u and v are adjacent points; point u and line e are incident with each other, as are v and e. A *walk* on a graph G is an alternating sequence of points and lines $v_0, e_1, v_1, e_2, \cdots, v_{n-1}, e_n, v_n$, beginning and ending with points, in which each line is incident with the two points immediately preceding and following it. A walk is called a *path* if all the points (and thus necessarily all the lines) are distinct. A graph is *connected* if every pair

2010 *Mathematics Subject Classification.* 05C25, 05C76.

Key words and phrases. Line Graph of Commuting Graph, Clique number, Chromatic number, Genus.

of points are joined by a path. The *length* of a walk is the number of occurences of lines in it. The *degree* of a point v_i in graph G, denoted by $deg_G(v_i)$, is the number of lines incident with v_i. The shortest $u-v$ path is often called a *geodesic*. The *diameter*, $diam(G)$ of a connected graph G is the length of any longest geodesic. A *clique* of a graph is a maximal complete subgraph. The maximum size of a clique in a graph G is called the *clique number* of G and is denoted by $\omega(G)$. A *colouring* of a graph is an assignment of colors to its points so that no two adjacent points have the same color. The *chromatic number* $\chi(G)$ is defined as the minimum n for which G has an n-colouring. A graph is *planar* if it can be embedded in the plane. The *genus* of a simple graph G is the smallest integer g such that G can be embedded into an orientable surface S_g. Since the number of vertices in $L(G)$ is the same as the number of edges in G, from the following theorem we have the number of vertices in $L(C(D_{2n}, D_{2n}))$.

Theorem 1.1. [4]*: For any integer* $n \geq 3$, *let* $G = C(D_{2n}, D_{2n})$. *Then the number of edges in* G,

$$\varepsilon(G) = \begin{cases} n\frac{(n+1)}{2} & if n \text{ is odd} \\ n\frac{(n+4)}{2} & otherwise. \end{cases}$$

The following lemmas are used in the proofs of our main results.

Lemma 1.1. [3]*: (Fundamental Theorem of Graph Theory)*
The sum of the degrees of the points of a graph G is twice the number of lines,

$$\sum deg(v_i) = 2q.$$

Lemma 1.2. [3]*: (Kuratowski's Theorem)*
A graph is planar if and only if it has no subgraph homeomorphic to k_5 *or* $K_{3,3}$.

Lemma 1.3. [3]*: For* $p \geq 3$, *the genus of the complete graph is*

$$\gamma(k_p) = \left\lceil \frac{(p-3)(p-4)}{12} \right\rceil.$$

2. MAIN RESULTS

Theorem 2.1. *Let* $n \geq 3$ *be an odd integer. Let* $G = L(C(D_{2n}, D_{2n}))$. *Then*

$i)$ $deg_G(\{1, sr^i\}) = 2n-2; 1 \leq i \leq n$

$ii)$ $deg_G(\{1, r^j\}) = 3n-4; 1 \leq j \leq n-1$

$iii)$ $deg_G(\{r^k, r^l\}) = 2n-4; k < l, 1 \leq k \leq n-2$ *and* $2 \leq l \leq n-1$.

Proof. Let $n \geq 3$ be an odd integer.

i) The vertex $\{1, sr^i\}$; $1 \leq i \leq n$ is adjacent with each of the vertices of the form $\{1, sr^j\}$; $j \neq i$, $1 \leq j \leq n$ and $\{1, r^t\}$; $1 \leq t \leq n-1$. Hence $deg_G(\{1, sr^i\}) = (n-1)+(n-1) = 2n-2$ for $1 \leq i \leq n$.

ii) The vertex $\{1,r^j\}$; $1\le j\le n-1$ is adjacent with each of the vertices of the form $\{1,r^k\}$; $k\ne j,n$, $1\le k\le n-1$, $\{1,sr^i\}$; $1\le i\le n$, $\{r^j,r^l\}$; $1\le l\le n-1$, $l\ne j,n$. Hence $deg_G(\{1,r^j\})=(n-2)+n+(n-2)=3n-4$ for $1\le j\le n-1$.

iii) The vertex $\{r^k,r^l\}$; $k<l$, $1\le k\le n-2$, $2\le l\le n-1$ is adjacent with each of the vertices of the form $\{r^k,r^s\}$; $s\ne k,l$, $1\le s\le n$ and $\{r^t,r^l\}$; $t\ne l,k$, $1\le t\le n$. Hence $deg_G(\{r^k,r^l\})=(n-2)+(n-2)=2n-4$ for $k<l$, $1\le k\le n-2$ and $2\le l\le n-1$. □

Theorem 2.2. *Let $n\ge 3$ be an even integer. Let $G=L(C(D_{2n},D_{2n}))$. Then*

i) $deg_G(\{1,sr^i\})=2n; 1\le i\le n$
ii) $deg_G(\{r^{\frac{n}{2}},sr^i\})=2n; 1\le i\le n$
iii) $deg_G(\{sr^i,sr^{i\oplus_n\frac{n}{2}}\})=4; 1\le i\le n$
iv) $deg_G(\{1,r^j\})=3n-4; j\ne\frac{n}{2}, 1\le j\le n-1$
v) $deg_G(\{1,r^{\frac{n}{2}}\})=4n-4$
vi) $deg_G(\{r^k,r^l\})=2n-4; k<l, 1\le k\le n-2, 2\le l\le n-1, k,l\ne n,\frac{n}{2}$
vii) $deg_G(\{r^k,r^{\frac{n}{2}}\})=3n-4; k\ne\frac{n}{2}, 1\le k\le n-1.$

Proof. Let $n\ge 3$ be an even integer.

i) The vertex $\{1,sr^i\}$; $1\le i\le n$ is adjacent with each of the vertices of the form $\{1,sr^t\}$; $t\ne i$, $1\le t\le n$, $\{r^{\frac{n}{2}},sr^i\}$, $\{sr^{i\oplus_n\frac{n}{2}},sr^i\}$ and $\{1,r^j\}$; $1\le j\le n-1$. Hence $deg_G(\{1,sr^i\})=(n-1)+1+1+(n-1)=2n$ for $1\le i\le n$.

ii) The vertex $\{r^{\frac{n}{2}},sr^i\}$; $1\le i\le n$ is adjacent with each of the vertices of the form $\{r^{\frac{n}{2}},sr^t\}$; $t\ne i$, $1\le t\le n$, $\{r^{\frac{n}{2}},r^j\}$; $j\ne\frac{n}{2}$, $1\le j\le n$ and $\{1,sr^i\}$ and $\{sr^i,sr^{i\oplus_n\frac{n}{2}}\}$. Hence $deg_G(\{r^{\frac{n}{2}},sr^i\})=(n-1)+(n-1)+1+1=2n$ for $1\le i\le n$.

iii) The vertex $\{sr^i,sr^{i\oplus_n\frac{n}{2}}\}$; $1\le i\le n$ is adjacent with each of the vertices $\{1,sr^t\}$; $t=i,i\oplus_n\frac{n}{2}$ and $\{r^{\frac{n}{2}},sr^t\}$; $t=i,i\oplus_n\frac{n}{2}$. Hence $deg_G(\{sr^i,sr^{i\oplus_n\frac{n}{2}}\})=4$ for $1\le i\le n$.

iv) The vertex $\{1,r^j\}$; $j\ne\frac{n}{2}$, $1\le j\le n-1$ is adjacent with each of the vertices of the form $\{1,r^t\}$; $t\ne j$, $1\le t\le n-1$ and $\{r^j,r^m\}$; $m\ne j$, $1\le m\le n-1$ and $\{1,sr^i\}$; $1\le i\le n$. Hence $deg_G(\{1,r^j\})=(n-2)+(n-2)+n=3n-4$ for $1\le j\le n-1$ and $j\ne\frac{n}{2}$.

v) The vertex $\{1,r^{\frac{n}{2}}\}$ is adjacent with each of the vertices of the form $\{1,r^m\}$; $m\ne\frac{n}{2}$, $1\le m\le n-1$, $\{r^{\frac{n}{2}},r^m\}$; $m\ne\frac{n}{2}$, $1\le m\le n-1$, $\{1,sr^i\}$; $1\le i\le n$ and $\{r^{\frac{n}{2}},sr^i\}$; $1\le i\le n$. Hence $deg_G(\{1,r^{\frac{n}{2}}\})=(n-2)+(n-2)+n+n=4n-4$.

vi) The vertex $\{r^k,r^l\}$; $k<l$, $1\le k\le n-2$, $2\le l\le n-1$, $k,l\ne\frac{n}{2}$ is adjacent with each of the vertices of the form $\{r^k,r^t\}$; $1\le t\le n$, $t\ne k,l$ and $\{r^u,r^l\}$; $1\le u\le n$, $u\ne k,l$. Hence $deg_G(\{r^k,r^l\})=(n-2)+(n-2)=2n-4$ for $k<l$, $1\le k\le n-2$, $2\le l\le n-1$, $k,l\ne n,\frac{n}{2}$.

vii) The vertex $\{r^k,r^{\frac{n}{2}}\}$; $k\ne\frac{n}{2}$; $1\le k\le n-1$ is adjacent with each of the vertices of the form $\{r^k,r^t\}$; $1\le t\le n$, $t\ne k,\frac{n}{2}$, $\{r^u,r^{\frac{n}{2}}\}$; $1\le u\le n$, $u\ne k,\frac{n}{2}$

and $\left\{sr^i, r^{\frac{n}{2}}\right\}$; $1 \le i \le n$. Hence $deg_G(\left\{r^k, r^{\frac{n}{2}}\right\}) = (n-2)+(n-2)+n = 3n-4$ for $k \neq \frac{n}{2}$, $1 \le k \le n-1$. □

Theorem 2.3. *Let $n \ge 3$ be any integer and let $G = L(C(D_{2n}, D_{2n}))$. Then the number of edges in G,*

$$E(G) = \begin{cases} \frac{n^3-n}{2} & if\ n\ is\ odd \\ \frac{n^3+3n^2+2n}{2} & if\ n\ is\ even. \end{cases}$$

Proof. **Case i:** n is odd
By the Fundamental theorem of Graph Theory and Theorem 2.1, we have

$$2n^2 - 2n + (n-1)(3n-4) + \left(\frac{n(n-3)}{2} + 1\right)(2n-4) = 2q$$

$$\Rightarrow q = \frac{n^3-n}{2}.$$

Case ii: n is even
By the Fundamental theorem of Graph Theory and Theorem 2.2, we have

$$2n^2 + 2n^2 + 2n + (n-2)(3n-4) + (4n-4) + \left(\frac{n^2-5n+6}{2}\right)(2n-4) + \\ + (n-2)(3n-4) = 2q$$

$$\Rightarrow q = \frac{n^3+3n^2+2n}{2}.$$ □

Theorem 2.4. *Let $n \ge 3$ be any integer and let $G = L(C(D_{2n}, D_{2n}))$. Then $\omega(G) = \chi(G) = 2n-1$.*

Proof. Consider the subset
$\alpha_1 = \left\{\{1,r\}, \left\{1,r^2\right\}, \cdots, \left\{1,r^{n-1}\right\}, \{1,sr\}, \left\{1,sr^2\right\}, \cdots, \{1,sr^n\}\right\}$.
Then $L(C(D_{2n}, \alpha_1))$ is a complete subgraph of G.
Claim: $L(C(D_{2n}, \alpha_1))$ is a clique of G.

Case i: n is odd
Consider the vertex $\left\{r^k, r^l\right\}$; $k < l$, $1 \le k \le n-2$, $2 \le l \le n-1$. Now $\left\{r^k, r^l\right\}$ is not adjacent with any of $\left\{1, sr^i\right\}$; $1 \le i \le n$. Thus the graph $L\left(C\left(D_{2n}, \alpha_1 \cup \left\{r^k, r^l\right\}\right)\right)$ is not complete.

Hence $M = L(C(D_{2n}, \alpha_1))$ is a clique of G, when n is odd and $|M| = 2n-1$.

Let M_1 be a maximum clique of G. Let $\{a,b\}$ and $\{c,d\}$ be any two vertices of M_1. $\{a,b\}$ and $\{c,d\}$ are adjacent when $a = c$ or $b = d$. When $a = c$, there are $2n-1$ such vertices and when $b = d$, there are $n-1$ such vertices. Hence $|M_1| = 2n-1$.

Case ii: n is even
Consider the vertex $\left\{r^k, r^l\right\}$; $k < l$, $1 \le k \le n-2$, $2 \le l \le n-1$. Now $\left\{r^k, r^l\right\}$ is not adjacent with any of $\left\{1, sr^i\right\}$; $1 \le i \le n$. Thus the graph $L\left(C\left(D_{2n}, \alpha_1 \cup \left\{r^k, r^l\right\}\right)\right)$ is not complete.
Consider the vertex $\left\{r^{\frac{n}{2}}, sr^i\right\}$; $1 \le i \le n$. Now $\left\{r^{\frac{n}{2}}, sr^i\right\}$ is not adjacent with any of $\left\{1, r^j\right\}$; $j \neq \frac{n}{2}$, $1 \le j \le n-1$. Thus the graph $L\left(C\left(D_{2n}, \alpha_1 \cup \left\{r^{\frac{n}{2}}, sr^i\right\}\right)\right)$ is not complete.

Consider the vertex $\left\{sr^i, sr^{i\oplus_n \frac{n}{2}}\right\}; 1 \leq i \leq n$. Now $\left\{sr^i, sr^{i\oplus_n \frac{n}{2}}\right\}$ is not adjacent with any of $\left\{1, r^j\right\}; 1 \leq j \leq n-1$. Thus the graph $L\left(C\left(D_{2n}, \alpha_1 \cup \left\{sr^i, sr^{i\oplus_n \frac{n}{2}}\right\}\right)\right)$ is not complete.

Hence $M = L(C(D_{2n}, \alpha_1))$ is a clique of G, when n is even and $|M| = 2n-1$.

Let M_2 be a maximum clique of G. Let $\{a,b\}$ and $\{c,d\}$ be any two vertices of M_1. $\{a,b\}$ and $\{c,d\}$ are adjacent when $a=c$ or $b=d$. When $a=c$, there are $2n-1$ such vertices and when $b=d$, there are $n+2$ such vertices. Hence $|M_2| = 2n-1$.

Hence $\omega(G) = 2n-1$.

Claim: $\chi(G) = 2n-1$

Since $\omega(G) = 2n-1$, $2n-1$ colours are required to colour the subgraph induced by α_1. Let $c(x)$ denote the colour of the vertex x where $x \in \alpha_1$.

Case i: n is even Ĺet $i<j$, $1 \leq i \leq n-2$ and $2 \leq j \leq n-1$. Then assign

$$c\left(\left\{r^i, r^j\right\}\right) = \begin{cases} c\left(\left\{1, r^{i\oplus_n j}\right\}\right) & if\ i+j \neq n \\ c\left(\left\{1, sr^i\right\}\right) & if\ i+j = n \end{cases}$$

Let $1 \leq k \leq n$. Then assign

$$c\left(\left\{r^{\frac{n}{2}}, sr^k\right\}\right) = \begin{cases} c\left(\left\{1, sr^{\frac{n}{2}\oplus_n k}\right\}\right) & if\ k+\frac{n}{2} \neq n \\ c\left(\left\{1, sr^{\frac{n}{2}+k}\right\}\right) & if\ k+\frac{n}{2} = n \end{cases}$$

and

$$c\left(\left\{sr^k, sr^{k\oplus_n \frac{n}{2}}\right\}\right) = c\left(\left\{1, r^t\right\}\right) \text{ for any } t \in \{1,2,3,\cdots,n-1\}$$

Case ii: n is odd

Let $i<j$, $1 \leq i \leq n-2$ and $2 \leq j \leq n-1$. Then assign

$$c\left(\left\{r^i, r^j\right\}\right) = \begin{cases} c\left(\left\{1, r^{i\oplus_n j}\right\}\right) & if\ i+j \neq n \\ c\left(\left\{1, sr^i\right\}\right) & if\ i+j = n. \end{cases}$$

□

Theorem 2.5. *Let $G = L(C(D_{2n}, D_{2n}))$, where $n \geq 3$ is any integer. Then*

$$diam(G) = \begin{cases} 2 & if\ n\ is\ odd \\ 3 & otherwise. \end{cases}$$

Proof. Let n be an odd integer. Let $\{a,b\}$, $\{c,d\}$ be any two vertices of G. If $\{a,b\}$ and $\{c,d\}$ are adjacent, then either $a=c$ or d or $b=c$ or d. Now, suppose $\{a,b\}$ and $\{c,d\}$ are not adjacent then $a \neq b \neq c \neq d$. In this case, $\{a,b\}$ is adjacent with $\{b,c\}$ which in turn is adjacent with $\{c,d\}$. Hence there exists a path of length 2. Hence $diam(G) = 2$, when n is odd.

But when n is even, the vertex $\left\{r^k, r^l\right\}; k<l, 1 \leq k \leq n-2, 2 \leq l \leq n-1, k,l \neq \frac{n}{2}$ is not adjacent with the vertices $\left\{sr^i, sr^{i\oplus_n \frac{n}{2}}\right\}; 1 \leq i \leq n$. In this case $\left\{r^k, r^l\right\}$ is adjacent with $\left\{r^{\frac{n}{2}}, r^t\right\}; t = k or l$ which in turn is adjacent with $\left\{r^{\frac{n}{2}}, sr^i\right\}; 1 \leq i \leq n$. Hence there exists a path of length 3. Hence $diam(G) = 3$, when n is even. □

Corollary 2.1. *Let $G = L(C(D_{2n}, \alpha))$, where α is any subset of the vertex set of $L(C(D_{2n}, D_{2n}))$.*

i) *If n is odd and* $\alpha = \{\{r^i, r^j\}, \{1, sr^k\} : 1 \leq i \leq n-2,\ 2 \leq j \leq n-1,\ 1 \leq k \leq n\}$, *then* $diam(G) = \infty$.

ii) *If n is even and* $\alpha = \{\{r^i, r^j\}, \{sr^i, sr^{i \oplus_n \frac{n}{2}}\} : 1 \leq i \leq n,\ 1 \leq j \leq n-1\}$, *then* $diam(G) = \infty$.

Theorem 2.6. *For* $n \geq 3$, *the line graph* $G = L(C(D_{2n}, D_{2n}))$ *is non-planar.*

Proof. For $n \geq 3$,
the induced subgraph $\langle \{\{1, sr\}, \{1, sr^2\}, \{1, sr^3\}, \{1, r\}, \{1, r^2\}\} \rangle$ is K_5.
Hence by Kuratowski's Theorem, $L(C(D_{2n}, D_{2n}))$; $n \geq 3$ is non-planar. □

Theorem 2.7. *For* $n = 3$, *the genus of the line graph* $G = L(C(D_6, D_6))$ *is 1.*

Proof. Let $G = L(C(D_6, D_6))$.
Then $V(G) = \{\{1, sr\}, \{1, sr^2\}, \{1, sr^3\}, \{1, r\}, \{1, r^2\}, \{r, r^2\}\}$.
The induced subgraph $\langle \{\{1, sr\}, \{1, sr^2\}, \{1, sr^3\}, \{1, r\}, \{1, r^2\}\} \rangle$ is K_5.
By Lemma 1.3, $\gamma(k_5) = 1$.
Thus $\gamma(G) \geq 1$. On the other hand, we can embed G into S_1 as shown in figure 2.1.
Therefore, $\gamma(G) = 1$.

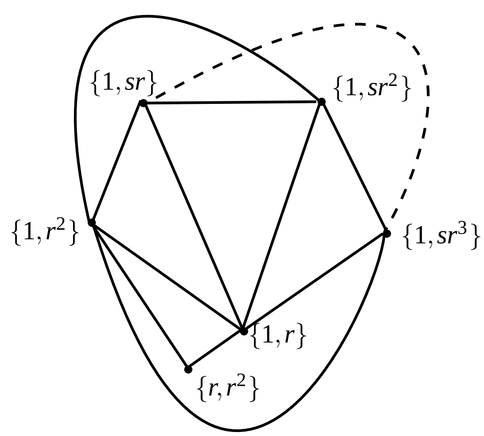

Figure 2.1 □

Theorem 2.8. *For* $n > 3$, *the lower bound for the genus of the line graph* $G = L(C(D_{2n}, D_{2n}))$ *is* $\left\lceil \frac{(2n-4)(2n-5)}{12} \right\rceil$.

Proof. Let $G = L(C(D_{2n}, D_{2n}))$.
Since $\omega(G) = 2n-1$, the genus of the graph G will be greater than $\gamma(K_{2n-1})$.
By Lemma 1.4, $\gamma(K_{2n-1}) = \left\lceil \frac{(2n-4)(2n-5)}{12} \right\rceil$. Therefore,

$$\gamma(G) \geq \left\lceil \frac{(2n-4)(2n-5)}{12} \right\rceil.$$

□

Theorem 2.9. *For $n > 3$, the upper bound for the genus of the line graph $G = L(C(D_{2n}, D_{2n}))$ is*

$$\gamma(G) \leq \begin{cases} \left\lceil \frac{n(n^3+2n^2-13n-14)}{48} \right\rceil & if\ n\ is\ odd \\ \left\lceil \frac{n(n^3+8n^2+2n-56)}{48} \right\rceil & if\ n\ is\ even. \end{cases}$$

Proof. Let $G = L(C(D_{2n}, D_{2n}))$.

Case i: n is odd

Since the number of vertices in G is $\frac{n(n+1)}{2}$ and the graph is not complete, the genus of the graph G will be less than $\gamma\left(K_{\frac{n(n+1)}{2}}\right)$. By Lemma 1.4, $\gamma\left(K_{\frac{n(n+1)}{2}}\right) = \left\lceil \frac{(n^2+n-6)(n^2+n-8)}{48} \right\rceil$. Therefore,

$$\gamma(G) < \left\lceil \frac{(n^2+n-6)(n^2+n-8)}{48} \right\rceil$$

Thus,

$$\gamma(G) \leq \left\lceil \frac{n(n^3+2n^2-13n-14)}{48} \right\rceil.$$

Case ii: n is even

Since the number of vertices in G is $\frac{n(n+4)}{2}$ and the graph is not complete, the genus of the graph G will be less than $\gamma\left(K_{\frac{n(n+4)}{2}}\right)$. By Lemma 1.4, $\gamma\left(K_{\frac{n(n+4)}{2}}\right) = \left\lceil \frac{(n^2+4n-6)(n^2+4n-8)}{48} \right\rceil$. Therefore,

$$\gamma(G) < \left\lceil \frac{(n^2+4n-6)(n^2+4n-8)}{48} \right\rceil.$$

Thus,

$$\gamma(G) \leq \left\lceil \frac{n(n^3+8n^2+2n-56)}{48} \right\rceil.$$

□

REFERENCES

[1] David S. Dummit and Richard M. Foote, *Abstract Algebra* (Second Edition), John Wiley and Son, Inc(Asia) Pvt. Ltd, Singapore (2005).

[2] F. Harary, *Graph Theory*, Addision-Wesley Reading M.A, 1969.

[3] Huadong Su and Pailing Li, *On the Genus of the Zero-Divisor Graph of $\mathbb{Z}_n$*, International Journal of Combinatorics Vol.2014, Article ID 390732, 5 pages (2014).

[4] T. Tamizh Chelvam, K. Selvakumar and S. Raja, *Commuting Graphs on Dihedral Group*, The Journal of Mathematics and Computer Science Vol.2, No.2(2011) 402-406.

(Received: September 22, 2019)
(Revised: October 20, 2021)

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