

REMEMBERING ACADEMICIAN MANOJLO MARAVIĆ



In mid-November 2019 it was one hundred years since the birth, and in May 2020, twenty years since the death of Manojlo Maravić, professor at the Faculty of Civil Engineering of the University of Sarajevo and member of the Academy of Sciences and Arts of Bosnia and Herzegovina (ANUBiH). On this occasion we devote this article to a brief retrospective of his life and his contribution to Mathematics.

Academician Manojlo Maravić was born on the 17th of November 1919 in Drežnica, Municipality of Ogulin (Croatia). He attended primary school in Drežnica and Ogulin, and high school in Karlovac.

Already in high school, he developed such an affection for mathematics that he decided to make of it a life-long study. Thus, immediately after graduating from high school, in 1938, he enrolled as a student of mathematics at the Faculty of Philosophy in Belgrade. The war that started in April 1941 caught the then student Manojlo Maravić in Belgrade. The University closed and he returned home to Drežnica. He joined the *People's Liberation War*, in which he participated from the beginning of 1941 until the liberation in 1945, and celebrated the end of the war as a JNA major and bearer of "*Partizanska Spomenica 1941*" (*Commemorative Medal of the Partisans of 1941*).

After the war he went back to Belgrade to continue his studies, which he completed in 1947. Recognized as an extremely talented and hard-working student, M. Maravić started his academic career immediately after graduation, as the first assistant at the *Department of Mathematics* of the newly established *Faculty of Natural Sciences and Mathematics* in Belgrade, for the course Complex Analysis, with the

celebrated professor, academician Jovan Karamata. At the same time, Maravić was appointed as an assistant at the *Mathematical Institute of the Serbian Academy of Sciences* (SAN).

In 1952 he moved to Sarajevo and, from October 1952 to June 1953, he worked as a professor at the *First Male Gymnasium* and was elected first as an assistant and soon after as an assistant professor at the then *Faculty of Technical Studies*. He received his doctorate in 1956 from the *Faculty of Philosophy* in Sarajevo, defending his doctoral dissertation *On a Method of Summability of Divergent Series* under the mentorship of a very important member of the *Belgrade School of Mathematics*, academician Vojislav G. Avakumović. In 1961, he was elected associate professor, and in 1967, full professor at the Faculty of Civil Engineering in Sarajevo.

In this brief article we can only offer a small glimpse at the results and papers of academician Manojlo Maravić.

Academician Maravić was not only one of the first teachers and the first scientists in the field of mathematics in Sarajevo and BiH, but also the first mathematician and the 13th scientist from BiH to obtain a doctorate at the University of Sarajevo.

His successful and rich academic career, which begun in the mid-fifties of the last century, was related, almost exclusively, to Sarajevo and the University of Sarajevo. In terms of his contribution to science and university teaching, he was one of the most prominent mathematicians in our country.

Prof. Maravić published around forty scientific papers. His scientific papers aroused notable interest both of Yugoslav scientists and those around the world and took place in two important and at that time very current areas of Mathematical Analysis: *Theory of Summability and Fourier Analysis*. With his doctoral dissertation, he began his research in the field of the Theory of Summability, to which he remained attached, whether through starting new research in this field or applying its results in other fields of mathematics. Another area in which he worked intensively and achieved significant results was Fourier Analysis and the related questions from the Theory of Differential Equations. Here he especially dealt with the applications of the Theory of Summability to the multiple Fourier series and the summability of the expansions of functions from the L^2 class by the eigenfunctions of the Laplacian operator in n -dimensional space.

The papers of M. Maravić listed in the attached list under numbers [1, 4, 5, 6, 8, 34], belong to the field of Theory of Summability. In these papers he obtained many very interesting results. Thus, for example, in paper [1], he proved a convexity theorem for the G-method of summation. It should be especially pointed out that there are only a few convexity theorems and that Maravić's was the second of two that were known at the time. The first such theorem was proved by the great Hungarian mathematician M. Riesz, for the summability method named after him the Riesz's summability method.

In his papers related to the G -summability of the expansions of functions from L^2 class, by the eigenfunctions of the Laplacian operator in n -dimensional space, M. Maravić has solved many important problems and among other things found necessary and sufficient conditions for which G -summability is a local property of a function at the point where it develops (see [2] and [3]). Thanks to the results obtained in these papers his work was noticed by the international community of mathematicians. This group of papers also includes papers [13, 17, 19, 21, 23], of which [17] and [23] should be especially noted, in which he studied the spectral function of the Laplace operator. In these papers, starting from the results of *Avakumović, Tichmarsh, Levitan, Avadhani, Minakshisundaram*, Maravić expanded numerous classical results and opened a number of completely new branches of research.

Academician Maravić also included his doctoral students in the solving of problems which he himself was working on: *M. Galić* (1974), *K. Finci* (1977), *S. Šlaković* (1978) and the author of this text *M. Vuković* (1979). In addition, at the time when he was a visiting professor at Wayne State University, he was a co-mentor of another doctoral dissertation along with the American mathematician *Daniel Waterman*.

Almost all other scientific papers of M. Maravić belong to the Theory of multiple Fourier series of class L^1 . Starting from the fundamental results of Bochner and Chandrasekharan, he formulated and proved a number of theorems in this area as well. Particularly notable is his result, obtained through a generalization of a basic Bochner theorem, in which he expressed Riesz's means of spherical partial sums of the multiple Fourier series of the observed function using its spherical means of higher order. In addition, in the case when the asymptotic behaviour of this mean is related to the behaviour of a Karamata's slowly oscillating function, M. Maravić obtained many results on the asymptotic behaviour of Riesz's, as well as generalized Bochner means of spherical partial sums of multiple Fourier series. Particularly interesting results, which he obtained in this area, are those in which the behaviour of spherical means of higher order stands in relation to the behaviour of one slowly oscillating function with the remainder term. He also obtained a series of such results for derivative multiple Fourier series. In his later years he published several papers on Riesz summability of a complex order of multiple Fourier series, thus successfully inserting himself in research founded by the American mathematician E. Stein.

Prof. Maravić also dedicated his scientific monograph "*Summability of development by Laplacian eigenfunctions in n -dimensional space*", published in the edition of "*Djela ANUBiH*" [M 1] to the scientific problems he dealt with. This monograph, like all his works, was extremely well presented in all three reference journals: *Mathematical Reviews* (USA), *Matematika of Academy of Sciences* (former USSR), and *Zentralblatt für Mathematik* (former FR Germany). A rather extensive

review of this monograph published by the *Zentralblatt für Mathematik, B. Crstici*, among other things, highlights the doctoral theses of the students of the "*Sarajevo Mathematical School formed around Academician Maravić*" [1979; Zbl. 0435.42001, 237-238].

In addition, in order to help students at a time when there was no textbook literature, academician Maravić wrote 2 textbooks: Mathematics I (for engineers) in co-authorship with academician B. Martić, and the *Collection of Problems in the Theory of Functions of one Complex Variable*, with very nice problems which academician J. Karamata gave as exam questions, at the time when Maravić was his assistant. These questions, written in his beautiful handwriting, still attract attention today by their beauty, originality, complexity and the elegance of proofs.

Professor Maravić published his scientific papers in highly reputable and respected national and international mathematical journals, and they were all very positively presented in the already mentioned world's most prestigious reference mathematical journals, and were cited in several world monographs, such as: K. Zeller – W. Beckmann, *Theorie der Limitierungsverfahren* and E. Seneta, *Regularly Varying Functions*, as well as in the publication of the *All-Union Institute of Scientific and Technical Information of the USSR Academy of Sciences: "Itogi nauki i tehniki", Mathematical Analysis series*, etc.

Thanks to his work and the results he obtained, M. Maravić drew the attention of the international scientific community, which resulted in the invitation from the famous American mathematician D. Waterman to be a visiting professor at Wayne State University (Detroit, USA), where he gave lectures for students and postgraduates during two academic years (1965/66 and 1966/67). Although he received an offer to stay in America permanently, Prof. Maravić did not accept it. Prior to that, through the Yugoslav Commission for Cultural Relations with Foreign Countries, he spent two academic years (1962/63 and 1963/64) in Sudan as a professor of mathematics at the University of Khartoum.

The renowned and respected academician Maravić was invited to give lectures on his scientific results at numerous departments of mathematics and mathematical institutes of universities and academies of sciences, both in the former Yugoslavia and abroad. We will mention only some of the foreign ones, such as the British universities from London: University of London and Woolwich Polytechnic University, and the American universities: Syracuse University, Syracuse, New York; Wayne State University, Detroit, Michigan; University of South Florida, Tampa, Florida; University of California (UCLA), Los Angeles, etc.

Prof. Maravić was undoubtedly one of the most important and versatile mathematicians in BiH and the former Yugoslavia. He significantly influenced the development of mathematics, both at the University of Sarajevo and throughout BiH.

Furthermore, he was also one of our top mathematicians who had a lot of understanding for the problems of teaching. As one of the key professors of mathematical analysis and its applications, through his scientific and pedagogical work, he gave clear contours to the *Sarajevo School of Mathematical Analysis*. It should be especially emphasized that he participated in the formation of the first postgraduate study of mathematics at the Department of Mathematics of the University of Sarajevo, of which he was also the first head.

Not only as a very active scientist and researcher, but also as an excellent and serious lecturer, always ready to help, Maravić had a strong influence on many generations of students and postgraduates, but also doctoral students, who admired and greatly appreciated him. Despite the fact that he was very strict, he was a favourite among students, because he was considered an extremely good and just professor.

This is confirmed by the quote from the article "*Mathematics as a song*" published in an interview on the occasion of the *July 27th Award*: "*Academician Professor Dr. Manojlo Maravić is like a balm for a sore wound. He is very objective. He simply proves to the student that they don't know something, he goes from one question to another, and there is nothing anyone can complain about*" – students of the Faculty of Civil Engineering wrote about their professor in a survey when asked if mathematics is difficult for them. (*I. Kalkan, Oslobođenje*, 21 May 1978).

As a natural result of rich scientific, professional and all other activities, Prof. Manojlo Maravić became a corresponding member of the ANUBiH in 1967, and a full member in 1975. He was also a member of the Scientific Society of Serbia.

Academician M. Maravić was constantly engaged in many other important tasks, such as serving as the Head of the *General Mathematics Division* of the *Mathematics Department of the Natural Sciences and Mathematics Faculty*, long time Head of the *Mathematics and Physic Department of Faculty of Civil Engineering* in Sarajevo, *President of the Society of Mathematicians, Physicists and Astronomers of BiH*, a member of the *Republic Council for the Improvement of Educational Work and Science* in BiH, as well as in the *Academy of Sciences and Arts of BiH*, where he performed many important duties, such as: *Secretary of the Department of Technical Sciences* and member of the *Presidency of ANUBiH*, *Editor-in-Chief* of journals "*Radovi Odjeljenja Tehničkih Nauka ANUBiH*" and "*Radovi Matematički ANU-BiH*", which today appears as the "*Sarajevo Journal of Mathematics*". Furthermore, he was a member of the Committee of the *Council of Academies for Cooperation with the American National Academy of Sciences* and a member of numerous committees in the Mathematics Department, Faculty, University, and ANUBiH.

For his outstanding contribution to the development of mathematics during his many years of work, academician Manojlo Maravić received a large number of high recognitions and many awards of which we mention only the few most important ones: *Society of Mathematicians, Physicists and Astronomers of Yugoslavia* (1974) as well as of *Bosnia and Herzegovina* (1977), and in the name of the president

of SFRJ comrade Tito "*Medal of Work with a Red Flag*" (1976), etc. In particular it must be mentioned that he received *the highest Republic Award for Science "Veselin Masleša"* (1968), the "*July 27 Award*" for scientific achievement (1975), the "*Sixth of April Award of the City of Sarajevo*" (1979), and the highest of all *ZAVNOBiH Award* (1988) – which only a few scientist from BiH have received.

This story would be incomplete if we didn't mention something about the personality of academician Manojlo Maravić. He was a very stable, established personality and a well-meaning man, of moderate and natural demeanour. He was a man of clear thoughts and developed attitudes, tolerant and unobtrusive, and at the same time a very witty and interesting storyteller. He was an intellectual who, in addition to mathematics, was an exceptional connoisseur of literature and foreign languages (he spoke English, and also used French, German and Russian), as well as history – a field in which he proved himself an exceptional author. Together with a group of his compatriots, he embarked on a serious venture – writing the history of his homeland – to pull it from the grip of oblivion. Thus, while participating in the writing of the book *Partizanska Drežnica* [OP 1], he covered the period from the settling of Serbs in the area of Drežnica to the disintegration of the Austro-Hungarian monarchy (1918), when his homeland became part of Yugoslavia. He used numerous data and written documents in his work, including the most reliable ones kept in the Vienna archives, which he obtained thanks to his high school professor of history and theology Milan Radeka.

Academician M. Maravić was a very prolific and distinguished scientific creator in the field of Mathematical Analysis. He was a scientist of general Yugoslav importance and high international reputation – a man who with his scientific work, teaching and mentoring contributed most to the formation of young scientists in mathematics in BiH. He was the author of a scientific monograph, one of the founders of the *School of Mathematical Analysis* in Sarajevo, a visiting professor and lecturer by invitation at numerous universities and institutes, a scientist of recognizable style of the Belgrade – Karamata school whose motto was: "*to get as many conclusions as possible with as few assumptions as possible, making sure that despite brevity, everything is crystal clear*".

Academician Manojlo Maravić – our dear Mane, an exceptional man and friend, a teacher of generations of students, a scientist who grew up with the University of Sarajevo – has left a deep mark on our environment. His friends and students still remember him today, 20 years after his death, with love and deep respect, and keep his name and work safe from oblivion.

It was a great privilege and honour to be a pupil of academician Manojlo Maravić who introduced us, his pupils, to the great, enchanting world of mathematical science: the world of *Divergent Series and their Convergence*, *Theory of Summability*, *Karamata's Theory*, and *Fourier Analysis and their Applications*.

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