RADOVI MATEMATIČKI
Vol. 13 (2004), 53–62

On some classes of bivariate functions characterized by formulas for the best approximation

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Abstract. This paper is devoted to calculation formulas for the best approximation of bivariate functions by sums of univariate functions. New classes of bivariate functions depending on some numerical parameter are constructed and characterized in terms of the best approximation calculation formulas.

1. Introduction

The approximation problem considered here is to approximate a continuous and real-valued function of two variables by the sum of two continuous functions of one variable. To make the problem more precise let Q be a compact set in the xy plane. Consider an approximation of a continuous function $f(x,y) \in C(Q)$ by the manifold $D = \{\varphi(x) + \psi(y)\}$, where $\varphi(x), \psi(y)$ are defined and continuous on the projections of Q onto the coordinate axes x and y respectively. The best approximation is defined as the distance from f to D :

$$E(f) = dist(f, D) = \inf_{D} \|f - \varphi - \psi\|_{C(Q)} = \inf_{D} \max_{(x,y)\in Q} |f(x,y) - \varphi(x) - \psi(y)|$$

A function $\varphi_0(x) + \psi_0(y)$ from D, if it exists, is called an extremal element or a best approximating sum if

$$E(f) = \|f - \varphi_0 - \psi_0\|_{C(Q)} \, .$$

To show that $E(f)$ depends also on Q, in some cases to avoid confusion, we write $E(f,Q)$ instead of $E(f)$.

In this paper we deal with calculation formulas for $E(f)$. In 1951 Diliberto and Straus published a paper [4] in which along with other results they established a formula for $E(f,R)$, where R here and throughout this paper is a rectangle with sides parallel to coordinate axes, containing the supremum

2000 Mathematics Subject Classification: 41A30, 41A50, 41A65.

over all closed lightning bolts (for this terminology see [1], [6]-[8]). Later the same formula was established by other authors differently in rectangular case (see [8]) and for more general sets (see [6], [7]). Although the formula is valid for all continuous functions, it is not easily calculable. Some authors have been seeking easily calculable formulas for the best approximation for some subsets of continuous functions. Rivlin and Sibner published a result [9] which allows one to find the exact value of $E(f, R)$ for a function $f(x, y)$ having a continuous and nonnegative derivative $\frac{\partial^2 f}{\partial x \partial y}$. This result in the more general case (for a function of n variables) was proved by Flatto [5]. Babaev [2] generalized Rivin and Sibner's result ([3] as well as Flatto's result) for a function $f(x, y)$ having nonnegative differencies $\Delta_{h_1,h_2} f$. More precisely he considered the class $M(R)$ of continuous functions $f(x, y)$ with the property

$$\Delta_{h_1,h_1} f = f(x, y) + f(x + h_1, y + h_2) - f(x, y + h_2) - f(x + h_1, y) \geq 0$$

for each rectangle $[x, x + h_1] \times [y, y + h_2] \subset R$, and proved that if $f(x, y)$ belongs to $M(R)$, where $R = [a_1, b_1] \times [a_2, b_2]$, then

$$E(f, R) = \frac{1}{4} \left[f(a_1, a_2) + f(b_1, b_2) - f(a_1, b_2) - f(b_1, a_2) \right].$$

As seen from this formula to calculate $E(f)$ it is sufficient to find only four values of $f(x, y)$ at the vertices of R. One can see that the formula also gives a sufficient condition for membership in the class $M(R)$, i.e. if

$$E(f, S) = \frac{1}{4} \left[f(x_1, y_1) + f(x_2, y_2) - f(x_1, y_2) - f(x_2, y_1) \right],$$

for a given f and for each $S = [x_1, x_2] \times [y_1, y_2] \subset R$, then the function $f(x, y)$ is in $M(R)$.

Our purpose is to construct new classes of continuous functions depending on a numerical parameter and characterize each class in terms of the best approximation calculation formulas. This parameter will show which points of R the calculation formula involves. We will also construct a best approximating sum $\varphi_0 + \psi_0$ to a function from the constructed classes.

2. Definition of the main classes

Let throughout this paper $R = [a_1, b_1] \times [a_2, b_2]$ be a rectangle and $c \in (a_1, b_1]$. Denote $R_1 = [a_1, c] \times [a_2, b_2]$ and $R_2 = [c, b_1] \times [a_2, b_2]$. It is clear that $R = R_1 \cup R_2$ and if $c = b_1$ then $R = R_1$.

We associate each rectangle $S = [x_1, x_2] \times [y_1, y_2]$ lying in R with the following functional:

$$L(f, S) = \frac{1}{4}\left[f(x_1, y_1) + f(x_2, y_2) - f(x_1, y_2) - f(x_2, y_1)\right].$$

Definition 2.1. *We say that a continuous function $f(x, y)$ belongs to the class $V_c(R)$ if*

1) $L(f, S) \geq 0$, *for each* $S \subset R_1$;
2) $L(f, S) \leq 0$, *for each* $S \subset R_2$;
3) $L(f, S) \geq 0$, *for each* $S = [a_1, b_1] \times [y_1, y_2]$, $\quad S \subset R$.

It can be shown that for any $c \in (a_1, b_1]$ the class $V_c(R)$ is not empty. Indeed, one can easily verify that the function

$$v_c(x, y) = \begin{cases} w(x, y) - w(c, y), & (x, y) \in R_1 \\ w(c, y) - w(x, y), & (x, y) \in R_2 \end{cases}$$

where $w(x, y) = \left(\frac{x - a_1}{b_1 - a_1}\right)^{\frac{1}{n}} \cdot y$ and $n \geq \log_2 \frac{b_1 - a_1}{c - a_1}$, satisfies conditions 1)-3) and therefore belongs to $V_c(R)$. The class $V_c(R)$ has the following obvious properties:

a) For given functions $f_1, f_2 \in V_c(R)$ and numbers $\alpha_1, \alpha_2 \geq 0$, $\alpha_1 f_1 + \alpha_2 f_2 \in V_c(R)$. $V_c(R)$ is a closed subset of the space of continuous functions.
b) $V_{b_1}(R) = M(R)$.
c) If f is a common element of $V_{c_1}(R)$ and $V_{c_2}(R)$, $a_1 < c_1 < c_2 \leq b_1$ then $f(x, y) = \varphi(x) + \psi(y)$ on the rectangle $[c_1, c_2] \times [a_2, b_2]$.

The properties a) and b) are clear. The property c) also becomes clear if we note that by definition of the classes $V_{c_1}(R)$ and $V_{c_2}(R)$ for each rectangle

$$S \subset [c_1, c_2] \times [a_2, b_2]$$

$$L(f, S) \leq 0 \text{ and } L(f, S) \geq 0$$

respectively. Hence

$$L(f, S) = 0 \text{ for each } S \subset [c_1, c_2] \times [a_2, b_2].$$

Then it is not difficult to see that f is of the form $\varphi(x) + \psi(y)$ on the rectangle $[c_1, c_2] \times [a_2, b_2]$.

Proposition 2.2. *If a function $f(x, y)$ has the continuous derivative $\frac{\partial^2 f}{\partial x \partial y}$ on the rectangle R which satisfies the following conditions*

1) $\frac{\partial^2 f}{\partial x \partial y} \geq 0$, *for all* $(x, y) \in R_1$;

2) $\frac{\partial^2 f}{\partial x \partial y} \leq 0$, *for all* $(x, y) \in R_2$;
3) $\frac{df(a_1,y)}{dy} \leq \frac{df(b_1,y)}{dy}$, *for all* $y \in [a_2, b_2]$,
then $f(x, y)$ *belongs to* $V_c(R)$.

The proof of this proposition is very simple and can be obtained by integrating both sides of the inequalities in conditions 1)-3) over the sets $[x_1, x_2] \times [y_1, y_2] \subset R_1$, $[x_1, x_2] \times [y_1, y_2] \subset R_2$ and $[y_1, y_2] \subset [a_2, b_2]$ respectively.

Example 2.3. Consider the function $f(x, y) = y \sin \pi x$ on the unit square $K = [0, 1] \times [0, 1]$ and rectangles $K_1 = \left[0, \frac{1}{2}\right] \times [0, 1]$, $K_2 = \left[\frac{1}{2}, 1\right] \times [0, 1]$. It is not difficult to verify that the considered function satisfies all conditions of the proposition and therefor belongs to $V_{\frac{1}{2}}(K)$.

3. The construction of an extremal element

Theorem 3.1 *The best approximation of a function* $f(x, y)$ *from the class* $V_c(R)$ *can be calculated by the formula*

$$E(f, R) = L(f, R_1) = \frac{1}{4} \left[f(a_1, a_2) + f(c, b_2) - f(a_1, b_2) - f(c, a_2) \right].$$

Let y_0 *be any solution from* $[a_2, b_2]$ *of the equation*

$$L(f, Y) = \frac{1}{2} L(f, R_1), \quad Y = [a_1, c] \times [a_2, y].$$

Then a function $\varphi_0(x) + \psi_0(y)$, *where*

$$\varphi_0(x) = f(x, y_0),$$

$$\psi_0(y) = \frac{1}{2} \left[f(a_1, y) + f(c, y) - f(a_1, y_0) - f(c, y_0) \right]$$

is a best approximating sum in the manifold D.

To prove the theorem we first prove

Lemma 3.2. *Let* $f(x, y)$ *be a function from* $V_c(R)$ *and* $X = [a_1, x] \times [y_1, y_2]$ *be a rectangle with fixed* $y_1, y_2 \in [a_2, b_2]$. *Then the function* $h(x) = L(f, X)$ *has the properties:*
1) $h(x) \geq 0$, *for any* $x \in [a_1, b_1]$;
2) $\max\limits_{[a_1,b_1]} h(x) = h(c)$ *and* $\min\limits_{[a_1,b_1]} h(x) = h(a_1) = 0$.

Proof. If $X \subset R_1$ then the validity of $h(x) \geq 0$ follows from the definition of $V_c(R)$. If X is from R but not lying in R_1 then by denoting $X' = [x, b_1] \times [y_1, y_2]$, $S = X \cup X'$ and using the obvious equality

$$L(f, S) = L(f, X) + L(f, X')$$

we deduce from the definition of $V_c(R)$ that $L(f, X) \geq 0$.

To prove the second part of the lemma it is enough to show that $h(x)$ increases in the interval $[a_1, c]$ and decreases in the interval $[c, b_1]$. Indeed, if $a_1 \leq x_1 \leq x_2 \leq c$ then

$$h(x_2) = L(f, X_2) = L(f, X_1) + L(f, X_{12}), \tag{1}$$

where $X_1 = [a_1, x_1] \times [y_1, y_2]$, $X_2 = [a_1, x_2] \times [y_1, y_2]$, $X_{12} = [x_1, x_2] \times [y_1, y_2]$. Taking into consideration that $L(f, X_1) = h(x_1)$ and X_{12} lies in R_1 we conclude from (1) that $h(x_2) \geq h(x_1)$.

If $c \leq x_1 \leq x_2 \leq b_1$ then X_{12} lies in R_2 and we conclude from (1) that $h(x_2) \leq h(x_1)$.

Proof of Theorem 3.1. It is obvious that $L(f, R_1) = L(f - \varphi - \psi, R_1)$ for every sum $\varphi(x) + \psi(y)$. Hence

$$L(f, R_1) \leq \|f - \varphi - \psi\|_{C(R_1)} \leq \|f - \varphi - \psi\|_{C(R)}.$$

As the sum $\varphi(x) + \psi(y)$ is arbitrary, $L(f, R_1) \leq E(f, R)$. To complete the proof it is sufficient to construct a sum $\varphi_0(x) + \psi_0(y)$ for which the equality

$$\|f - \varphi_0 - \psi_0\|_{C(R)} = L(f, R_1) \tag{2}$$

holds. Consider the function

$$g(x, y) = f(x, y) - f(x, a_2) - f(a_1, y) + f(a_1, a_2).$$

The function g has the following obvious properties

1) $g(x, a_2) = g(a_1, y) = 0$;
2) $L(f, R_1) = L(g, R_1) = \frac{1}{4} g(c, b_2)$;
3) $E(f, R) = E(g, R)$;
4) The function of one variable $g(c, y)$ increases in the interval $[a_2, b_2]$.

The last property of g lets us write that

$$0 = g(c, a_2) \leq \frac{1}{2} g(c, b_2) \leq g(c, b_2).$$

Since $g(x, y)$ is continuous, there exists at least one solution $y = y_0$ of the equation

$$g(c, y) = \frac{1}{2} g(c, b_2)$$

or, using different notation

$$L(f, Y) = \frac{1}{2} L(f, R_1), \quad \text{where } Y = [a_1, c] \times [a_2, y].$$

Denote

$$\varphi_1(x) = g(x, y_0),$$

$$\psi_1(y) = \frac{1}{2}\left(g(c, y) - g(c, y_0)\right),$$

$$G(x, y) = g(x, y) - \varphi_1(x) - \psi_1(y).$$

Calculate the norm of $G(x, y)$ on R. Consider the rectangles $R' = [a_1, b_1] \times [y_0, b_2]$ and $R'' = [a_1, b_1] \times [a_2, y_0]$. It is clear that

$$\|G\|_{C(R)} = \max\left\{\|G\|_{C(R')}, \|G\|_{C(R'')}\right\}.$$

First calculate the norm $\|G\|_{C(R')}$:

$$\|G\|_{C(R')} = \max_{(x,y)\in R'} |G(x, y)| = \max_{y\in[y_0,b_2]} \max_{x\in[a_1,b_1]} |G(x, y)|. \tag{3}$$

For a fixed point y (we keep it fixed until (6)) from the interval $[y_0, b_2]$

$$\max_{x\in[a_1,b_1]} G(x, y) = \max_{x\in[a_1,b_1]} \left(g(x, y) - g(x, y_0)\right) - \psi_1(y) \tag{4}$$

and

$$\max_{x\in[a_1,b_1]} G(x, y) = \min_{x\in[a_1,b_1]} \left(g(x, y) - g(x, y_0)\right) - \psi_1(y). \tag{5}$$

By Lemma 3.2 the function

$$h_1(x) = 4L(f, X) = g(x, y) - g(x, y_0), \text{ where } X = [a_1, x] \times [y_0, y],$$

reaches its maximum on $x = c$ and minimum on $x = a_1$:

$$\max_{x\in[a_1,b_1]} h_1(x) = g(c, y) - g(c, y_0)$$

$$\min_{x\in[a_1,b_1]} h_1(x) = g(a_1, y) - g(a_1, y_0) = 0.$$

Using these facts in (4) and (5) we obtain

$$\max_{x\in[a_1,b_1]} G(x, y) = g(c, y) - g(c, y_0) - \psi_1(y) = \frac{1}{2}\left(g(c, y) - g(c, y_0)\right),$$

$$\min_{x\in[a_1,b_1]} G(x, y) = -\psi_1(y) = -\frac{1}{2}\left(g(c, y) - g(c, y_0)\right).$$

Consequently

$$\max_{x\in[a_1,b_1]} |G(x, y)| = \frac{1}{2}\left(g(c, y) - g(c, y_0)\right). \tag{6}$$

Taking (6) and the 4^{th} property of g into account in (3) we obtain

$$\|G\|_{C(R')} = \frac{1}{2}\left(g(c,b_2) - g(c,y_0)\right) = \frac{1}{4}g(c,b_2).$$

Analogously it can be shown that

$$\|G\|_{C(R'')} = \frac{1}{4}g(c,b_2).$$

Hence

$$\|G\|_{C(R)} = \frac{1}{4}g(c,b_2) = L(f,R_1).$$

But by the definition of G

$$G(x,y) = g(x,y) - \varphi_1(x) - \psi_1(y) = f(x,y) - \varphi_0(x) - \psi_0(y),$$

where

$$\varphi_0(x) = \varphi_1(x) + f(x,a_2) - f(a_1,a_2) + f(a_1,y_0) = f(x,y_0),$$

$$\psi_0(y) = \psi_1(y) + f(a_1,y) - f(a_1,y_0) = \frac{1}{2}\left(f(a_1,y) + f(c,y) - f(a_1,y_0) - f(c,y_0)\right).$$

So,

$$\|f - \varphi_0 - \psi_0\|_{C(R)} = L(f,R_1).$$

Hence we have proved (2) and thus Theorem 3.1. Notice that the function $\varphi_0(x) + \psi_0(y)$ is a best approximating sum in the manifold D .

Remark 3.3. In special case $c = b_1$ Theorem 3.1 turns into Babaev's result from [2].

Corollary 3.4. *Let a function $f(x,y)$ have a continuous derivative $\frac{\partial^2 f}{\partial x \partial y}$ on the rectangle R and satisfy the following conditions*

1) $\frac{\partial^2 f}{\partial x \partial y} \geq 0$, *for all* $(x,y) \in R_1$;
2) $\frac{\partial^2 f}{\partial x \partial y} \leq 0$, *for all* $(x,y) \in R_2$;
3) $\frac{df(a_1,y)}{dy} \leq \frac{df(b_1,y)}{dy}$, *for all* $y \in [a_2,b_2]$.

Then

$$E(f,R) = L(f,R_1) = \frac{1}{4}\left[f(a_1,a_2) + f(c,b_2) - f(a_1,b_2) - f(c,a_2)\right].$$

The proof can be obtained from Proposition 2.2 and Theorem 3.1.

Remark 3.5. Rivlin and Sibner [9] proved Corollary 3.4 in the special case when $c = b_1$.

Example 3.6. As we have seen in Example 2.3 the function $f = y \sin \pi x$ belongs to $V_{\frac{1}{2}}(K)$, where $K = [0,1] \times [0,1]$. By theorem 3.1. $E(f,K) = \frac{1}{4}$ and the function $\frac{1}{2} \sin \pi x + \frac{1}{2} y - \frac{1}{4}$ is a best approximating sum.

Until now we have been approximating a function $f(x,y)$ from $V_c(R)$ on the rectangle R by the manifold D. As is seen from the following theorem in some cases the formula in Theorem 3.1 holds for more general sets than rectangles.

Theorem 3.7. *Let $f(x,y)$ be a function from $V_c(R)$ and $Q \subset R$ be a compact set which contains all vertices of R_1 (points $(a_1,a_2),(a_1,b_2),(c,a_2)$, (c,b_2)). Then*

$$E(f,Q) = L(f,R_1) = \frac{1}{4}\left[f(a_1,a_2) + f(c,b_2) - f(a_1,b_2) - f(c,a_2)\right].$$

Proof. As $Q \subset R$, $E(f,Q) \leq E(f,R)$. On the other hand by Theorem 3.1 $E(f,R) = L(f,R_1)$. Hence $E(f,Q) \leq L(f,R_1)$. It can be shown, as in the proof of Theorem 3.1, that $L(f,R_1) \leq E(f,Q)$. But then automatically $E(f,Q) = L(f,R_1)$.

Example 3.8. Calculate the best approximation of the function $f(x,y) = -(x-2)^{2n} y^m$ (n and m are positive integers) on the domain

$$Q = \left\{(x,y) : 0 \leq x \leq 2, 0 \leq y \leq (x-1)^2 + 1\right\}.$$

It can be easily verified that $f(x,y) \in V_2(R)$, where $R = [0,4] \times [0,2]$. Besides, Q contains all vertices of $R_1 = [0,2] \times [0,2]$. Consequently, by Theorem 3.7 $E(f,Q) = L(f,R_1) = 2^{2(n-1)+m}$.

4. Characterization of $V_c(R)$

Theorem 4.1. *The following conditions are necessary and sufficient for a continuous function $f(x,y)$ to be in $V_c(R)$:*

1) $E(f,S) = L(f,S)$, for each rectangle $S = [x_1,x_2] \times [y_1,y_2]$, $S \subset R_1$;

2) $E(f,S) = -L(f,S)$, for each rectangle $S = [x_1,x_2] \times [y_1,y_2]$, $S \subset R_2$;

3) $E(f,S) = L(f,S_1)$, for each rectangle $S = [a_1,b_1] \times [y_1,y_2]$, $S \subset R$ and where $S_1 = [a_1,c] \times [y_1,y_2]$.

In other words conditions 1)-3) characterize the class $V_c(R)$ in terms of the best approximation calculation formulas.

Proof. The necessity can be easily obtained by the definition of $V_c(R)$, Babaev's previously mentioned result (see Introduction) and Theorem 3.1. The sufficiency is clear if we observe that $E(f,S) \geq 0$.

As seen in the proof of Theorem 4.1 it is obvious, but it is an important and interesting quality, that the best approximation calculation formulas themselves define the class of approximated functions.

5. Classes $V_c^-(R), U(R)$ and $U^-(R)$

By $V_c^-(R)$ denote the class of such functions $f(x,y)$ that $-f$ is from $V_c(R)$. It is clear that $E(f,R) = -L(f,R_1)$, for each $f \in V_c^-(R)$.

We define $U_c(R), a_1 \le c < b_1$, as a class of continuous functions $f(x,y)$ with the properties

1) $L(f,S) \le 0$, for each rectangle $S = [x_1,x_2] \times [y_1,y_2]\,,\ S \subset R_1$;
2) $L(f,S) \ge 0$, for each rectangle $S = [x_1,x_2] \times [y_1,y_2]\,,\ S \subset R_2$;
3) $L(f,S) \ge 0$, for each rectangle $S = [a_1,b_1] \times [y_1,y_2]\,,\ S \subset R$.

Using the same techniques in the proof of Theorem 3.1 it can be shown that the following theorem is valid:

Theorem 5.1. *The best approximation of a function $f(x,y)$ from the class $U_c(R)$ can be calculated by the formula*

$$E(f,R) = L(f,R_2) = \frac{1}{4}\left[f(c,a_2) + f(b_1,b_2) - f(c,b_2) - f(b_1,a_2)\right].$$

Let y_0 be any solution from $[a_2,b_2]$ of the equation

$$L(f,Y) = \frac{1}{2}L(f,R_2), \qquad Y = [c,b_1] \times [a_2,y]\,.$$

Then a function $\varphi_0(x) + \psi_0(y)$, where

$$\varphi_0(x) = f(x,y_0), \quad \psi_0(y) = \frac{1}{2}\left[f(c,y) + f(b_1,y) - f(c,y_0) - f(b_1,y_0)\right],$$

is a best approximating sum from the manifold D to an f.

By $U_c^-(R)$ denote the class of such functions $f(x,y)$ that $-f$ is from $U_c(R)$. It is clear that $E(f,R) = -L(f,R_2)$, for each $f \in U_c^-(R)$.

Remark 5.2. The correspondingly modified versions of Theorems 3.7, 4.1 and Corollary 3.4 are also true for classes $V_c^-(R), U_c(R)$ and $U_c^-(R)$. They are obvious and can be formulated by the readers.

Example 5.3. Consider the function $f(x,y) = \left(x - \frac{1}{2}\right)^2 y$ on a unit square $K = [0,1] \times [0,1]$. It can be easily verified that $f \in U_{\frac{1}{2}}(K)$. So, by Theorem 5.1 $E(f,K) = \frac{1}{16}$ and $\frac{1}{2}\left(x - \frac{1}{2}\right)^2 + \frac{1}{8}y - \frac{1}{16}$ is a best approximating function.

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(Received: March 26, 2004)

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O nekim klasama dvovarijantnih funkcija karakteriziranih formulama za najbolju aproksimaciju

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Sadržaj

U radu se izračunavaju formule za najbolju aproksimaciju dvovarijantnih funkcija pomoću suma jednovarijantnih funkcija. Konstruirane su nove klase dvovarijantnih funkcija zavisne od nekih numeričkih parametara, koje su karakterizirane preko formula za izračunavanje najbolje aproksimacije.