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Results on the error function and the neutrix convolution [1]

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Abstract. Some neutrix convolutions of the error function erf(x) and its associated functions $\mathrm{erf}_+(x)$ and $\mathrm{erf}_-(x)$ with x^r, x_+^r and x_-^r are evaluated. Further neutrix convolutions are deduced.

The *error function* erf(x), see for example Sneddon [6], is the locally summable function defined by

$$\mathrm{erf}(\mathrm{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\mathrm{x}} \exp(-\mathrm{u}^2)\,\mathrm{du}. \tag{1}$$

More generally erf(λx) was defined in [4] for $\lambda \neq 0$ by

$$\mathrm{erf}(\lambda\mathrm{x}) = \frac{2}{\sqrt{\pi}} \int_0^{\lambda\mathrm{x}} \exp(-\mathrm{u}^2)\,\mathrm{du}. \tag{2}$$

It is easily seen that erf(x) is an odd function of x.

The locally summable functions $\mathrm{erf}_+(\lambda\mathrm{x})$ and $\mathrm{erf}_-(\lambda\mathrm{x})$ were defined for $\lambda \neq 0$ by

$$\mathrm{erf}_+(\lambda\mathrm{x}) = \mathrm{H}(\mathrm{x})\mathrm{erf}(\lambda\mathrm{x}) \quad \mathrm{erf}_-(\lambda\mathrm{x}) = \mathrm{H}(-\mathrm{x})\mathrm{erf}(\lambda\mathrm{x}),$$

where H denotes Heaviside's function. Note that

$$\mathrm{erf}_+[\lambda(-\mathrm{x})] = -\mathrm{erf}_-(\lambda\mathrm{x}), \quad \mathrm{erf}_-[\lambda(-\mathrm{x})] = -\mathrm{erf}_+(\lambda\mathrm{x}). \tag{3}$$

If f and g are locally summable functions then the classical convolution $f * g$ of f and g is given by

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)\,dt = \int_{-\infty}^{\infty} f(x-t)g(t)\,dt$$

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for all values of x for which the integral exists. It follows easily that if $f*g$ exists, then

$$f*g = g*f,$$
$$(f*g)' = f*g' = f'*g.$$

Before proving our results on the convolution, we need the following easily proved lemma:

Lemma 1.

$$\begin{aligned}\alpha_{2r}(x) &= \int_0^x t^{2r}\exp(-t^2)\,dt \\ &= -\sum_{i=0}^{r-1}\frac{(2r)!(r-i)!}{2^{2i+1}r!(2r-2i)!}x^{2r-2i-1}\exp(-x^2) + \frac{(2r)!\sqrt{\pi}}{2^{2r+1}r!}\,\mathrm{erf(x)}, \quad (4)\\ \alpha_{2r+1}(x) &= \int_0^x t^{2r+1}\exp(-t^2)\,dt \\ &= -\sum_{i=0}^{r}\frac{r!}{2(r-i)!}x^{2r-2i}\exp(-x^2) + \frac{r!}{2} \quad (5)\end{aligned}$$

for $r = 0, 1, 2, \ldots,$ *where the sum in* (4) *is empty when* $r = 0$.

We now let $\mathcal{D}$ be the space of infinitely differentiable functions with compact support and let $\mathcal{D}'$ be the space of distributions defined on $\mathcal{D}$.

Definition 1. The *convolution* $f*g$ of two distributions f and g in $\mathcal{D}'$ is defined by the equation

$$\langle (f*g)(x), \varphi\rangle = \langle f(y), \langle g(x), \varphi(x+y)\rangle\rangle$$

for arbitrary φ in $\mathcal{D}$, provided f and g satisfy either of the conditions

(a) either f or g has bounded support,
(b) the supports of f and g are bounded on the same side,

see Gel'fand and Shilov [5].

Note that if f and g are locally summable functions satisfying either of the above conditions and the classical convolution $f*g$ exists, then it is in agreement with Definition 1.

This definition of the convolution is rather restrictive and so the non–commutative neutrix convolution was introduced in [2]. In order to define the neutrix convolution product we first of all let τ be a function in $\mathcal{D}$ satisfying the following properties:

(i) $\tau(x) = \tau(-x)$,
(ii) $0 \le \tau(x) \le 1$,
(iii) $\tau(x) = 1$ for $|x| \le \frac{1}{2}$,
(iv) $\tau(x) = 0$ for $|x| \ge 1$.

The function τ_n is now defined by

$$\tau_n(x) = \begin{cases} 1, & |x| \le n, \\ \tau(n^n x - n^{n+1}), & x > n, \\ \tau(n^n x + n^{n+1}), & x < -n, \end{cases}$$

for $n = 1, 2, \ldots$.

Definition 2. Let f and g be distributions in $\mathcal{D}'$ and let $f_n = f\tau_n$ for $n = 1, 2, \ldots$. Then the *neutrix convolution* $f \circledast g$ is defined as the neutrix limit of the sequence $\{f_n * g\}$, provided that the limit h exists in the sense that

$$\underset{n\to\infty}{\text{N}-\lim}\langle f_n * g, \varphi\rangle = \langle h, \varphi\rangle,$$

for all φ in $\mathcal{D}$, where N is the neutrix, see van der Corput [1], having domain $N' = \{1, 2, \ldots, n, \ldots\}$ and range N'' the real numbers, with negligible functions finite linear sums of the functions

$$n^\lambda \ln^{r-1} n,\ \ln^r n \quad (\lambda > 0,\ r = 1, 2, \ldots)$$

and all functions which converge to zero in the usual sense as n tends to infinity.

Note that in this definition the convolution $f_n * g$ is defined in Gel'fand and Shilov's sense, the distribution f_n having bounded support. Note also that because of the lack of symmetry in the definition of $f \circledast g$, the neutrix convolution is in general non-commutative.

The following theorem was proved in [2], showing that the neutrix convolution is a generalization of the convolution.

Theorem 1. *Let f and g be distributions in $\mathcal{D}'$ satisfying either condition (a) or condition (b) of Gel'fand and Shilov's definition. Then the neutrix convolution $f \circledast g$ exists and*

$$f \circledast g = f * g.$$

The next theorem was also proved in [2].

Theorem 2. *Let f and g be distributions in $\mathcal{D}'$ and suppose that $f \circledast g$ exists, then the neutrix convolution $f \circledast g'$ exists and*

$$(f \circledast g)' = f \circledast g'. \tag{6}$$

Note however that $(f \circledast g)'$ is not necessarily equal to $f' \circledast g$. We do however have the following lemma which was proved in [3].

Lemma 2. *Let f and g be distributions in $\mathcal{D}'$ and suppose that $f \circledast g$ exists. If $\operatorname*{N-lim}_{n\to\infty}\langle (f\tau_n') * g, \varphi\rangle$ exists and equals $\langle h, \varphi\rangle$ for all φ in $\mathcal{D}'$, then the neutrix convolution $f' \circledast g$ exists and*

$$(f \circledast g)' = f' \circledast g + h. \tag{7}$$

In order to define further neutrix convolution products, we increase our set of negligible functions given in Definition 2 to also include finite linear sums of the functions

$$n^r \mathrm{erf}(\lambda \mathrm{n}), \quad \mathrm{r} = 1, 2, \dots, \quad \lambda \neq 0.$$

The following theorem was proved in [4].

Theorem 3. *The function $\langle n^r \mathrm{erf}[\lambda(\alpha \pm \mathrm{n})], \varphi\rangle$ is a negligible function for $r = 1, 2, \dots,$ $\lambda \neq 0$ and all φ in $\mathcal{D}$.*

We now prove

Theorem 4. *If $\lambda \neq 0$, then the neutrix convolution $\mathrm{erf}(\lambda \mathrm{x}) \circledast \mathrm{x}_+^\mathrm{r}$ exists and*

$$\mathrm{erf}(\lambda \mathrm{x}) \circledast \mathrm{x}_+^\mathrm{r} = \frac{2}{(r+1)\sqrt{\pi}} \sum_{i=0}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \alpha_i(\lambda x) x^{r-i+1} + \\ + \frac{2}{(r+1)\sqrt{\pi}} \sum_{i=1}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \beta_i x^{r-i+1} \tag{8}$$

for $r = 0, 1, 2, \dots$.

Proof. We put $\mathrm{erf_n}(\lambda \mathrm{x}) = \mathrm{erf}(\lambda \mathrm{x})\tau_\mathrm{n}(\mathrm{x})$ for $n = 1, 2, \dots$. Since $\mathrm{erf_n}(\lambda \mathrm{x})$ has compact support, the classical convolution $\mathrm{erf_n}(\lambda \mathrm{x}) * \mathrm{x}_+^\mathrm{r}$ exists and

$$\frac{\sqrt{\pi}}{2} \mathrm{erf_n}(\lambda \mathrm{x}) * \mathrm{x}_+^\mathrm{r} = \int_{-n}^{x} (x-t)^r \mathrm{erf}(\lambda \mathrm{t})\, \mathrm{dt} + \\ + \int_{-n-n^{-n}}^{-n} (x-t)^r \tau_n(t) \mathrm{erf}(\lambda \mathrm{t})\, \mathrm{dt} \\ = I_1 + I_2. \tag{9}$$

It is easily seen that

$$\lim_{n\to\infty} I_2 = 0. \tag{10}$$

Further,

$$\begin{aligned}
I_1 &= \int_{-n}^{x} (x-t)^r \int_0^{\lambda t} \exp(-u^2)\,du\,dt \\
&= \int_0^{\lambda x} \exp(-u^2) \int_{u/\lambda}^{x} (x-t)^r\,dt\,du - \int_{-\lambda n}^{0} \exp(-u^2) \int_{-n}^{u/\lambda} (x-t)^r\,dt\,du \\
&= \frac{1}{r+1} \int_0^{\lambda x} (x-u/\lambda)^{r+1} \exp(-u^2)\,du + \\
&\quad + \frac{1}{r+1} \int_{-\lambda n}^{0} [(x-u/\lambda)^{r+1} - (x+n)^{r+1}] \exp(-u^2)\,du \\
&= \frac{1}{r+1} \sum_{i=0}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \alpha_i(\lambda x) x^{r-i+1} + \\
&\quad - \frac{1}{r+1} \sum_{i=1}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \alpha_i(-\lambda n) x^{r-i+1} + \\
&\quad + \frac{\sqrt{\pi}}{2(r+1)} \sum_{i=1}^{r+1} \binom{r+1}{i} x^{r-i+1} n^i \operatorname{erf}(-\lambda \mathrm{n}).
\end{aligned}$$

Noting that

$$\lim_{n\to\infty} \alpha_{2i}(-\lambda n) = -\frac{(2i)!\sqrt{\pi}}{2^{2i+1} i!} \operatorname{sgn}\lambda, \quad \lim_{n\to\infty} \alpha_{2i+1}(-\lambda n) = \frac{i!}{2}$$

for $i = 0, 1, 2, \ldots$ and using Theorem 3, it follows that

$$\begin{aligned}
\mathrm{N}\!-\!\lim_{n\to\infty} I_1 &= \frac{1}{r+1} \sum_{i=0}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \alpha_i(\lambda x) x^{r-i+1} + \\
&\quad + \frac{1}{r+1} \sum_{i=1}^{r+1} \binom{r+1}{i} (-\lambda)^{-i} \beta_i x^{r-i+1}.
\end{aligned} \tag{11}$$

Equation (8) now follows from equations (9), (10) and (11).

Corollary 4.1. *If* $\lambda \neq 0$*, then the neutrix convolution* $\mathrm{erf}(\lambda \mathrm{x}) \circledast \mathrm{x}_-^{\mathrm{r}}$ *exists and*

$$\begin{aligned}
\mathrm{erf}(\lambda \mathrm{x}) \circledast \mathrm{x}_-^{\mathrm{r}} &= \frac{2(-1)^r}{(r+1)\sqrt{\pi}} \sum_{i=0}^{r+1} \binom{r+1}{i} \lambda^{-i} (-1)^{i+1} \alpha_i(\lambda x) x^{r-i+1} + \\
&\quad + \frac{2(-1)^r}{(r+1)\sqrt{\pi}} \sum_{i=1}^{r+1} \binom{r+1}{i} \lambda^{-i} \beta_i x^{r-i+1}
\end{aligned} \tag{12}$$

for $r = 0, 1, 2, \ldots$.

Proof. It follows from equations (4) and (5) that

$$\alpha_i(-x) = (-1)^{i+1}\alpha_i(x).$$

Equation (12) now follows on replacing x by $-x$ in equation (8).

Corollary 4.2. *If* $\lambda \neq 0$*, then the neutrix convolutions* $\exp(-\lambda^2x^2) \circledast x_+^r$ *and* $\exp(-\lambda^2x^2) \circledast x_-^r$ *exist and*

$$\begin{aligned}\exp(-\lambda^2x^2) \circledast x_+^r = {} & \frac{1}{r+1}\sum_{i=0}^{r+1}\binom{r+1}{i}(-\lambda)^{-i}\alpha_i'(\lambda x)x^{r-i+1}+ \\ & -\frac{1}{r+1}\sum_{i=1}^{r}\binom{r+1}{i+1}(i+1)(-\lambda)^{-i-1}\alpha_i(\lambda x)x^{r-i}+ \\ & -\frac{1}{r+1}\sum_{i=0}^{r}\binom{r+1}{i+1}(i+1)(-\lambda)^{-i-1}\beta_i x^{r-i}, \qquad (13)\end{aligned}$$

$$\begin{aligned}\exp(-\lambda^2x^2) \circledast x_-^r = {} & \frac{(-1)^{r+1}}{r+1}\sum_{i=0}^{r+1}\binom{r+1}{i+1}(-\lambda)^{-i}\alpha_i'(\lambda x)x^{r-i+1}+ \\ & -\frac{(-1)^r}{r+1}\sum_{i=1}^{r}\binom{r+1}{i+1}(i+1)(-1)^i\lambda^{-i-1}\alpha_i(\lambda x)x^{r-i}+ \\ & +\frac{(-1)^r}{r+1}\sum_{i=0}^{r}\binom{r+1}{i+1}(i+1)\lambda^{-i-1}\beta_i x^{r-i} \qquad (14)\end{aligned}$$

for $r = 0, 1, 2, \ldots$.

Proof. With $x < n + n^{-n}$, we have

$$\begin{aligned}[\mathrm{erf}(\lambda \mathrm{x})\tau_\mathrm{n}'(\mathrm{x})] * \mathrm{x}_+^\mathrm{r} = {} & \int_{-n-n^{-n}}^{-n} (x-t)^r \mathrm{erf}(\lambda \mathrm{t})\, \mathrm{d}\tau_\mathrm{n}(\mathrm{t}) \\ = {} & (x+n)^r \mathrm{erf}(-\lambda \mathrm{n}) - \frac{2}{\sqrt{\pi}}\int_{-\mathrm{n}-\mathrm{n}^{-\mathrm{n}}}^{-\mathrm{n}} (\mathrm{x}-\mathrm{t})^\mathrm{r} \exp(-\lambda^2\mathrm{t}^2)\tau_\mathrm{n}(\mathrm{t})\, \mathrm{dt} + \\ & + r\int_{-n-n^{-n}}^{-n} (x-t)^{r-1}\mathrm{erf}(\lambda \mathrm{t})\tau_\mathrm{n}(\mathrm{t})\, \mathrm{dt} \\ = {} & I_1 + I_2 + I_3. \qquad (15)\end{aligned}$$

It is clear that

$$\underset{n\to\infty}{\mathrm{N-lim}}\, I_1 = \lim_{n\to\infty} x^r \mathrm{erf}(-\lambda \mathrm{n}) = -\operatorname{sgn}\lambda\, \mathrm{x}^\mathrm{r}, \tag{16}$$

$$\lim_{n\to\infty} I_2 = I_3 = 0 \tag{17}$$

and it follows from equations (15), (16) and (17) that

$$\underset{n\to\infty}{\mathrm{N-lim}}\, [\mathrm{erf}(\lambda\mathrm{x})\tau'_\mathrm{n}(\mathrm{x})] * \mathrm{x}_+^\mathrm{r} = -\operatorname{sgn}\lambda\, \mathrm{x}^\mathrm{r}. \tag{18}$$

Differentiating equation (8) partially with respect to x and using Lemma 2, Theorem 3 and equation (18) gives equation (13). Equation (14) follows similarly from equation (12).

Corollary 4.3 *If* $\lambda \neq 0$, *then the neutrix convolutions* $[x\exp(-\lambda^2 x^2)] \circledast x_+^r$ *and* $[x\exp(-\lambda^2 x^2)] \circledast x_-^r$ *exist and*

$$\begin{aligned}
[x\exp(-\lambda^2 x^2)] \circledast x_+^r = {} & \frac{1}{r+1}\sum_{i=0}^{r+1}\binom{r+1}{i}(-\lambda)^{-i}\alpha'_i(\lambda x)x^{r-i+2} + \\
& + \frac{1}{r+1}\sum_{i=1}^{r+1}\binom{r+1}{i} i(-\lambda)^{-i-1}\alpha_i(\lambda x)x^{r-i+1} + \\
& + \frac{1}{r+1}\sum_{i=1}^{r+1}\binom{r+1}{i} i(-\lambda)^{-i-1}\beta_i x^{r-i+1},
\end{aligned} \tag{19}$$

$$\begin{aligned}
[x\exp(-\lambda^2 x^2)] \circledast x_-^r = {} & \frac{(-1)^{r+1}}{r+1}\sum_{i=0}^{r+1}\binom{r+1}{i}(-\lambda)^{-i}\alpha'_i(\lambda x)x^{r-i+2} + \\
& - \frac{(-1)^r}{r+1}\sum_{i=1}^{r+1}\binom{r+1}{i} i(-\lambda)^{-i-1}\alpha'_i(\lambda x)x^{r-i+1} + \\
& - \frac{(-1)^r}{r+1}\sum_{i=1}^{r+1}\binom{r+1}{i} i\lambda^{-i-1}\beta_i x^{r-i+1}
\end{aligned} \tag{20}$$

for $r = 0, 1, 2, \dots$.

Proof. Equation (19) follows on differentiating equation (8) partially with respect to λ. Equation (20) follows similarly from equation (12).

Corollary 4.4 *If* $\lambda \neq 0$, *then the neutrix convolution* $\mathrm{erf}(\lambda\mathrm{x}) \circledast \mathrm{x}^\mathrm{r}$ *exists and*

$$\mathrm{erf}(\lambda\mathrm{x}) \circledast \mathrm{x}^\mathrm{r} = -\frac{\operatorname{sgn}\lambda}{\mathrm{r}+1}\sum_{\mathrm{i}=0}^{[(\mathrm{r}+1)/2]}\binom{\mathrm{r}+1}{2\mathrm{i}}\frac{(2\mathrm{i})!}{2^{2\mathrm{i}-1}\mathrm{i}!}\lambda^{-2\mathrm{i}}\mathrm{x}^{\mathrm{r}-2\mathrm{i}+1} \tag{21}$$

for $r = 0, 1, 2, \ldots,$ *where* $[a]$ *denotes the integer part of* a.

Proof. Noting that $x^r = x_+^r + (-1)^r x_-^r$, it follows from equations (8) and (12) that

$$\begin{aligned}\mathrm{erf}(\lambda \mathrm{x}) \circledast \mathrm{x}^{\mathrm{r}} &= \frac{2}{(r+1)\sqrt{\pi}} \sum_{i=0}^{r+1} \binom{r+1}{i} [1+(-1)^i] \lambda^{-i} \beta_i x^{r-i+1} \\ &= \frac{4}{(r+1)\sqrt{\pi}} \sum_{i=0}^{[(r+1)/2]} \binom{r+1}{2i} \lambda^{-2i} \beta_{2i} x^{r-2i+1}\end{aligned}$$

and equation (21) follows.

Theorem 5. *If* $\lambda \neq 0$, *then the neutrix convolution* $[x^s \exp(-\lambda^2 x^2)] \circledast x^r$ *exists and*

$$\begin{aligned}&[x^{2s} \exp(-\lambda^2 x^2)] \circledast x^r = \\ &\quad = -\frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1} \sum_{i=0}^{[(r+1)/2]} \binom{r+1}{2i+1} \frac{(2i+1)(2s+2i)!}{2^{2s+2i}(s+i)!} \lambda^{-2s-2i-1} x^{r-2i}, \quad (22) \\ &[x^{2s+1} \exp(-\lambda^2 x^2)] \circledast x^r = \\ &\quad = \frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1} \sum_{i=0}^{[(r+1)/2]} \binom{r+1}{2i} \frac{i(2s+2i)!}{2^{2s+2i-1}(s+i)!} \lambda^{-2s-2i-1} x^{r-2i+1}, \quad (23)\end{aligned}$$

for $r, s = 0, 1, 2, \ldots$.

Proof. With $x < n + n^{-n}$, we have

$$\begin{aligned}[t^s \exp(-\lambda^2 x^2) \tau_n'(x)] * x_+^r &= \int_{-n-n^{-n}}^{-n} (x-t)^r t^s \exp(-\lambda^2 t^2)\, d\tau_n(t) \\ &= (x+n)^r (-n)^s \exp(-\lambda^2 n^2) \\ &\quad - \int_{-n-n^{-n}}^{-n} [(x-t)^r t^s \exp(-\lambda^2 t^2)]' \tau_n(t)\, dt\end{aligned}$$

and it follows easily that

$$\lim_{n \to \infty} [t^s \exp(-\lambda^2 x^2) \tau_n'(x)] * x_+^r = 0. \qquad (24)$$

Differentiating equation (21) partially with respect to x and using the Lemma and equation (24) now gives equation (22) for the case $s = 0$.

Now suppose that equation (22) holds for some s. Then differentiating equation (22) partially with respect to λ we get

$$[x^{2s+2}\exp(-\lambda^2x^2)] \circledast x^r =$$
$$= -\frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1}\sum_{i=0}^{[(r+1)/2]}\binom{r+1}{2i+1}\frac{(2i+1)(2i+2s)!}{2^{2s+2i+1}(s+i)!}(2s+2i+1)\lambda^{-2s-2i-3}x^{r-2i}$$
$$= -\frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1}\sum_{i=0}^{[(r+1)/2]}\binom{r+1}{2i+1}\frac{(2i+1)(2i+2s+2)!}{2^{2s+2i+2}(s+i+1)!}\lambda^{-2s-2i-3}x^{r-2i}$$

and equation (22) follows by induction.

Next, differentiating equation (21) partially with respect to λ gives equation (23) for the case $s = 0$.

Now suppose that equation (23) holds for some s. Then differentiating equation (23) partially with respect to λ we get

$$[x^{2s+3}\exp(-\lambda^2x^2)] \circledast x^r =$$
$$= \frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1}\sum_{i=0}^{[(r+1)/2]}\binom{r+1}{2i}\frac{i(2s+2i)!}{2^{2s+2i}(s+i)!}(2s+2i+1)\lambda^{-2s-2i-3}x^{r-2i+1}$$
$$= \frac{\sqrt{\pi}\,\mathrm{sgn}\,\lambda}{r+1}\sum_{i=0}^{[(r+1)/2]}\binom{r+1}{2i}\frac{i(2i+2s+2)!}{2^{2s+2i+1}(s+i+1)!}\lambda^{-2s-2i-3}x^{r-2i+1}$$

and equation (23) follows by induction.

For further related results, see [4].

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Rezultati o funkciji greške i neutrix konvoluciji

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Sadržaj

U radu se izračunavaju neke neutrix konvolucije funkcije greške erf(x) i njenih pridruženih funkcija $\mathrm{erf}_+(x)$ i $\mathrm{erf}_-(x)$ sa x^r, x_+^r i x_-^r. Daljnje neutrix konvolucije su izvedene.