

OBITUARY



Academician Fikret Vajzović
1928-2017

It is with great sadness that we inform our readers that the eminent mathematician and Academician Fikret Vajzović, former Professor at the Faculty of Natural and Mathematics Sciences at the University of Sarajevo and Professor Emeritus at the University of Sarajevo, an editor of the Sarajevo Journal of Mathematics, our respected and beloved professor and teacher, and, above all, a remarkable friend and human being, passed away on the 4th of November, 2017 in Sarajevo.

Fikret Vajzović belonged to the first generations of students of mathematics at the University of Sarajevo and became one of the most recognised and appreciated professors of the University. His teaching career at the University lasted almost four decades and his mathematical career never stopped—he remained mathematically active almost to the end of his life. Prof. Vajzović developed at the same time as his University and it is with his very significant and important contribution that he witnessed the growth of the University and mathematics in general in Sarajevo and, more generally, in Bosnia and Herzegovina. His involvement both in science and in the development of the mathematical studies at the University made Prof.

Vajzović one of the most significant contributors to the university in Sarajevo and wider.

Academician Fikret Vajzović was born on the 1st of May 1928 in Konjic, where he graduated from the primary school and then, after losing his father at an early age, he moved with his mother and his younger brother and sister to Mostar, the birthplace of his mother. It is there that he went to the secondary school and the Teachers School, from which he graduated in 1947. Immediately afterwards he took a job as a teacher and at the same time enrolled as a student of mathematics and physics at the Higher Teaching Academy in Sarajevo, from which he graduated in 1955. Then, while continuing his work as teacher, he entered the Department of Mathematics at the Faculty of Philosophy in Sarajevo, as a part-time student, where he obtained his degree in 1959.

Immediately after finishing his undergraduate studies, Fikret Vajzović was nominated Assistant at the Faculty of Philosophy and then at the newly formed Faculty of Natural and Mathematical Sciences. So from then, until his retirement in 1999, he served the Faculty of Natural and Mathematical Sciences—with the exception of the period 1976–1979, when he helped the nascent University “*Džemal Bijedić*” as a Professor at their Faculty of Mechanical Engineering, while continuing his professorial duties at the University of Sarajevo.

During the period between 1962 and 1964, Fikret Vajzović studied for his Master’s degree in Mathematics at the Department of Mathematics of the University of Zagreb, where he graduated as a Master of Mathematical Sciences in March 1965 with his Master’s thesis, entitled “*Funkcionalne jednačbe u vektorskim prostorima*” (“*Functional equations in vector spaces*”). Just a few months later, in September 1965, he defended his Ph. D. thesis under the supervision of the celebrated Professor Svetozar Kurepa, who has, in spite of a large number of students spread all over the world, always thought of Fikret Vajzović as his best student.

The professorial career of Fikret Vajzović started in 1965, when he was named a Docent (Assistant Professor). In 1972 he became an Associate Professor and finally, in 1979, a Full Professor at the University of Sarajevo.

Prof. Vajzović’s scientific activity was concerned with two important parts of Mathematical Analysis: Functional Analysis and the Theory of Functional Equations. His scientific articles touched various areas, such as: Summability, Theory of Functions, Measure Theory, Operator Theory on Hilbert and Banach Spaces, Banach algebras and Banach spaces, Semigroups in Fréchet spaces, etc.

It can certainly be said that Prof. Vajzović was one of the founders of modern mathematical analysis and mathematics in general in this region, as well as one of the best mathematicians ever to be associated with the Department of Mathematics and Bosnia and Herzegovina. He was the founder of the Sarajevo School of Functional Analysis.

Prof. Fikret Vajzović published around 40 scientific papers, two textbooks and he wrote lecture notes for the majority of subjects that he was teaching. The quality of these is such that they could be published in state. His articles are published in best journals of the former Yugoslavia, as well as in the international journals. All his papers are carefully reviewed in all three main international review journals, namely *Mathematical Reviews* in the USA, *Matematika Akademii Nauk* in the SSSR and *Zentralblatt für Mathematik* in Germany. The significance of mathematical results can be measured by their inclusion in important monographs, for example the volume *Functional equations in single variable of Monografie Matematyczne* by Marek Kuczma, (Warsaw, Polish Academy of Sciences, 1968), includes all the papers published up to that point by Fikret Vajzović. All his articles in the subject of functional analysis up to 1969 feature in the bibliography of the well-known journal "*Aequationes mathematicae*" (1969), which is published as a supplement to the monograph "*Vorlesungen ber Funktionalgleichungen und ihre Anwendungen*" by J. Aczel. Let us mention some of the most significant mathematical results obtained by Fikret Vajzović. For example, in his paper " *$S^{\alpha,\beta}$ -inclusion of summability methods*" he proved an inclusion theorem for $S^{\alpha,\beta}$ -summability methods and in this way, removed obstacles for a further development of that theory. He also proved the Krejn hypothesis (M. G. Krejn was a well-known Russian mathematician) connected with the spectre of entire operators in Hilbert spaces. Of course, it should be emphasised that the results of F. Vajzović had more than a theoretical importance. For example, in his joint work with K. Suruliz he solved a problem posed by E. D. Cashwell and C. I. Everett and whose solution gives as a corollary the solution of the Maxwell-Jüttner equation which is used in the theory of elastic collision of the molecules of the balanced idealised gas. This was the first exact solution of that equation. In his joint work with R. Vugdalić in the theory of integral group operators, F. Vajzović generalised the results of the celebrated Russian mathematician V. P. Maslov. He also proved various exponential formulas for α -integrated operator semigroups. Fikret Vajzović defined the notion of quasi-Hilbert space and, in joint work with his doctoral students, he investigated the properties of such spaces, the properties of groups and semigroups of operators

and the cosine operator functions on such spaces. They specifically studied Hilbert transform on quasi-Hilbert spaces. In his joint work with A. Šahović, Vajzović proved that the Hilbert transform H in quasi-Hilbert spaces is basically equal to $-AA_+^{-1}$ ($= -A_+A^{-1}$), where iA is the infinitesimal generator of the isometry group that determines the given Hilbert transform and A_+ is the positive root of the operator $-A^2$. It is important to notice that this was the first time that the Hilbert transform on quasi-Hilbert spaces was put in connection with the operators A and A_+ . The same authors proved further more that when the spectre of the operator A is positive, then $H = iI$, where i and I are the imaginary unit and the identity operator, respectively. Thanks to this surprising result one obtains a significant generalisation and completion of an earlier result by the renowned German mathematician D. Hilbert in the space L_2 , and the method also gives a new proof of a result by the well-known Hungarian mathematician M. Riesz for spaces L_p , when $1 < p < \infty$. Various local and international mathematicians used results of Fikret Vajzović, either directly or as a basis for further investigation. We shall only mention some of them: K. Lajkó refers to F. Vajzović in the title of his paper: “*On the Solution of a Functional Equation of F. Vajzović*“ (1972). He is further cited in the joint paper Z. Daroczy–K. Lajkó–L. Szekelyhidi “*Functional Equations on ordered fields*“ (1977), and in the work of J. A. Baker and I. Cioranescu. Among those who used the mathematical work of the Academician Fikret Vajzović are also his numerous students, at the master or doctoral level. Prof. Vajzović supervised around 50 B. Sc. theses, 17 Master’s theses and, finally, 8 doctoral theses. He also supervised two theses that were started in Moscow under the supervision of the well-known Russian mathematician A. G. Kostjučenko, with whom Prof. Vajzović collaborated personally during a year-long visit at the celebrated Lomonosov University. Many of Prof. Vajzović papers are obtained in collaboration with junior mathematicians, his master’s and doctoral students. Academician Vajzović was very cultured in mathematics and more generally, with well-defined scientific and professional criteria, a man of exceptional human qualities and an excellent lecturer. He was a refined intellectual who, in addition to mathematics, knew much about literature, classical music, philosophy and history. His entire contribution over the long period of activity is of irreplaceable importance to the life of the Department of Mathematics, the Faculty of Natural and Mathematical Sciences, the University of Sarajevo, as well as all over Bosnia and Herzegovina and beyond. Both by his scientific and by his pedagogic work, the Academician Fikret Vajzović made an enormous contribution to the development of scientific research and the formation

of pedagogical and scientific strength in the mathematical life in BiH. One has to emphasise his contribution in raising the quality and level of the mathematical education both at the undergraduate and the postgraduate level, where he taught for over 30 years.

During his long career, Fikret Vajzović obtained many awards, which recognised his contribution to the development of mathematics and mathematical life in BiH. Some of the most important recognitions were the BiH prize for scientific development, “*Veselin Masleša*“, obtained in 1979, the prize “*27 of July*“ in 1987, and, of course, the nomination to ANUBiH: he became a corresponding member in 1991 and a full member in 1995.

Saying our farewell to Prof. Vajzović, let us remark that he lived peacefully and with characteristic modesty, always wanting to learn and know and that he generously shared his knowledge with others. He was always ready to help those who needed him. This is, above all, the reason why he left such a mark everywhere where he worked and lived, at the University of Sarajevo, but also at the Universities of Mostar, Bihać and East Sarajevo.

Remembering his fatherly presence, his generosity and his excellent lectures that never left ambiguities in his students’ minds, we shall keep his memory alive and do our best to follow in his steps.

Editors in Chief:

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Corresponding Member ANUBiH

Prof. Dr. Mirna Džamonja
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Research publication

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